

That's the Way the Ball Bounces

In this activity you will be using the graphing calculator and CBR to gather data which is best modeled by a non-linear function.

Within the activity you and your group will be given specific directions about how to set up the experiment and what actions to take with the data you gather - it is VERY important that you follow these directions carefully or you may end up working with misleading data values. Also, DO NOT round the values produced by the CBR or calculator during this experiment; doing so may result in incorrect conclusions.

What you need: A well inflated rubber ball,
 a CBR,
 two (2) TI graphing calculators,
 link cable

What you are trying to do: Find a function that will model the relationship between the amount of time a ball has been bouncing and the height of the bounces

Anticipatory Questions:

1) What are the two values your function will relate?

2) What do you know about those values?

3) What variables does your group intend to use to represent those values?

Name: _____

4) What are some classes of non-linear functions? Which do you suspect will be the best model for the data you will gather? Why?

Activity - Part 1

Gather the supplies you need. Find a clear space where you can gather your data. You will need approximately 4 feet of clear, flat, hard ground to conduct your experiment.

Clear the lists on both calculators as well as any equations stored under $\boxed{Y=}$. Connect one of the calculators to the CBR and run the RANGER program.

At the program's main menu, choose option 1 "SETUP/SAMPLE"

Adjust the settings as shown:

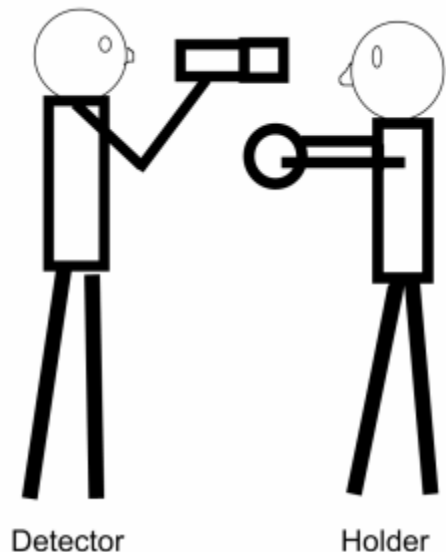
REALTIME: YES
TIME(S) : 15
DISPLAY: DIST
BEGIN ON: [TRIGGER]
SMOOTHING: LIGHT
UNITS: FEET

Hand the tallest member of your group the CBR and linked graphing calculator.

The CBR should be held at eye level with the sensor fully extended and aimed at the ground. This member of the group will be called "the Detector".

The second member of the group will hold the ball approximately 15 inches below the sensor and prepare to drop it straight down. We will call this member of the group "the Holder."

The starting positions for this experiment are shown to the right.

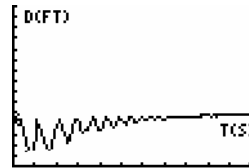
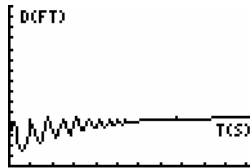


Name: _____

When everyone is in position, the Detector pushes the [TRIGGER] on the CBR and tells the Holder to drop the ball.

For the next 15 seconds the CBR will record the distance from the sensor to the ball. During this time, the Detector must be very careful to keep CBR at eye level and directly over the ball as it bounces.

When the CBR is done recording, examine the plot and ensure it is similar to one of the graphs below.



If your data produced a graph similar to those above with at least 5 visible "bumps" to it, then you have completed Part 1. If not, consult your teacher and repeat Part 1 if necessary.

Initial Observations:

- 5) After seeing the shape of the graph from Part 1, what class of non-linear function do you anticipate will provide the best model?

- 6) Describe what is happening on the right side of the graph.

- 7) Are you surprised the graph does not end with line on the x-axis? If the graph did hit the x-axis where is the ball at that instant? How do you know?

Activity - Part 2

It is now necessary for us to prepare the data for analysis.

The calculator connected to the CBR currently has all of the distance and time measurements recorded during your experiment saved in lists L1 and L2: the time at which each measurement was made is recorded in L1 and the distance the ball was from the sensor at each of those instants is recorded in L2.

You may recall we are looking for a model which relates the amount of time a ball has been bouncing and the height of the bounces - two quantities related to but different from the data we currently have stored.

The values in L1 are the amount of time that has passed since the Detector pressed the [TRIGGER]. We must adjust these values to take into account any delay by the Holder in dropping the ball.

The values in L2 are the distance between the CBR and the ball, not the ball and the ground. These values too must be adjusted.

Exit the RANGER program. Go to the [STAT PLOT] menu by pushing $\boxed{2nd}$ $\boxed{Y=}$. Turn on STAT PLOT 1 with the settings shown below:

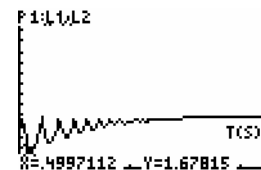
```

Plot1 Plot2 Plot3
Off Off
Type: L1 L2 L3 L4 L5 L6 L7 L8 L9 L0
Xlist: L1
Ylist: L2
Mark: +

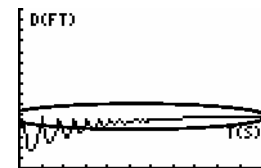
```

Push \boxed{ZOOM} $\boxed{9}$ to select [ZOOMSTAT] and then press the $\boxed{\square}$ button and move the cursor until you have identified the x-value of the deepest part of the first valley in the graph:

Write this x-value down. (DO NOT ROUND!)



Continue to trace to determine the highest y-value on the graph. It will likely fall within an area that corresponds to the the circled region in the graph at right.



Write that y-value down (DO NOT ROUND!), and press $\boxed{2nd}$ \boxed{MODE} to exit the graph.

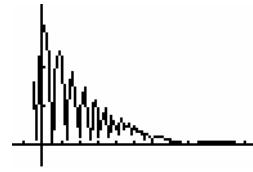
Now enter the following keystrokes to adjust your L1 values so that 0 seconds corresponds to when the ball first hit the ground, and adjust your L2 values to represent the height of the ball at those times.

Name: _____

L1 \square (the x-value you wrote down) $\text{STO} \blacktriangleright$ L1 ENTER
and

(the y-value you wrote down) \square L2 $\text{STO} \blacktriangleright$ L2 ENTER

Push ZOOM \square to select [ZOOMSTAT] and then press the TRACE button again. You should now have a graph that looks a bit more like a ball bouncing on the ground.



If your data produces a graph similar to that above with at least 5 visible "bounces" to it, then you have completed Part 2. If not, consult your teacher.

Checkpoint Questions:

- 8) After seeing the shape of the graph created by the adjusted L1 and L2 values, what non-linear function do you anticipate will provide the best model?

- 9) What do the x-values of the points in your current graph represent? What do the y-values of the points in your current graph represent? Using the variables you designated in Question 3, which variable corresponds to x and which to y on the graph?

Activity - Part 3

The goal of this activity is to create an equation which models the relationship between the height of a bounce and the time when its peak occurs.

To use regression to create that equation, you must identify the points which show the peak heights and the times which they occur must be identified.

On the calculator that was connected to the CBR push ZOOM \square to select [ZOOMSTAT] and then press the TRACE button. Move through the graph to identify the x and y coordinates of the peak of each bounce. Pay close attention to the subtle changes in the y-values at that far right of the graph to identify the relative peaks.

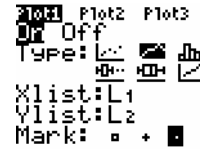
Name: _____

As each relative high spot is identified, it should be recorded both on paper and on the OTHER graphing calculator - the one which has not yet been used in the activity other than to clear its lists, will refer to this as Calculator2.

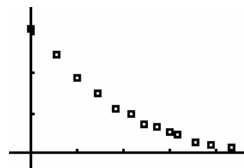
Enter each ordered pair on Calculator2, putting the UNROUNDED x-coordinates in L1 and the UNROUNDED y-coordinates of those points in L2.

Once you have constructed the lists on Calculator2, go to the [STAT PLOT] menu by pushing $\boxed{2nd} \boxed{Y=}$.

Turn on STAT PLOT 1 with the settings shown to the right:



Push $\boxed{ZOOM} \boxed{9}$ to select [ZOOMSTAT] and then press the \boxed{TRACE} button to ensure your points are successfully entered.



Your graph should look something like this:

If everything looks good, it is time to use the regression tools on your calculator to create modeling equations. If you believe your graph is not correct, consult your teacher.

On Calculator2, press \boxed{STAT} and move right to activate the [CALC] menu.

Use regression to create a quadratic models by choosing menu option 5 and keying L1,L2, Y1 . (L1, L2, Y1 tells the calculator how to reconstruct the ordered pairs and where you'd like the equation you created to be stored.) Be sure to use the L1 and L2 keys and choose Y1 from the [Y VARS] menu.

Press \boxed{STAT} to access the [CALC] menu again, this time creating a cubic model and storing it as Y2 by choosing menu option 6 and keying L1,L2,Y2 \boxed{ENTER} . Once more, be sure to use the L1 and L2 keys and choose Y2 from the [Y VARS] menu.

Access the \boxed{STAT} menu a third time and create an exponential model and storing it as Y3. Choose menu option 0 and keying L1,L2, Y3 .

Press $\boxed{Y=}$ to view the three equations you created, and answer the questions below.

Evaluation and Summary:

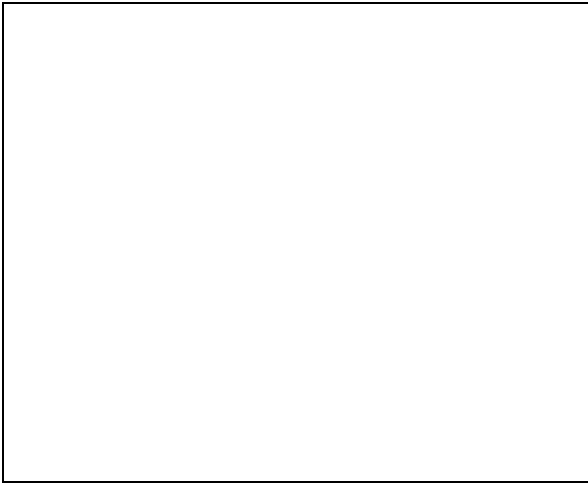
10) Write the quadratic equation you have stored as Y1.

Name: _____

- 11) Press $\boxed{Y=}$ and move the cursor over the = in the Y2 equation. Press $\boxed{\text{ENTER}}$. Repeat this process for Y3 so that the = in the Y1 equation is the only one highlighted. [See figure to the right] Press $\boxed{\text{GRAPH}}$.

```
Plot1 Plot2 Plot3
Y1=X^2
Y2=X^3
Y3=5^X
Y4=
Y5=
Y6=
Y7=
```

Sketch your graph in the box below.



- 12) Write the cubic equation you have stored as Y2.

- 13) Press $\boxed{Y=}$ and move the cursor over the = in the Y1 equation. Press $\boxed{\text{ENTER}}$. Repeat this process for Y2 so that the = in the Y2 equation is the only one highlighted. Press $\boxed{\text{GRAPH}}$.

Sketch your graph in the box below.

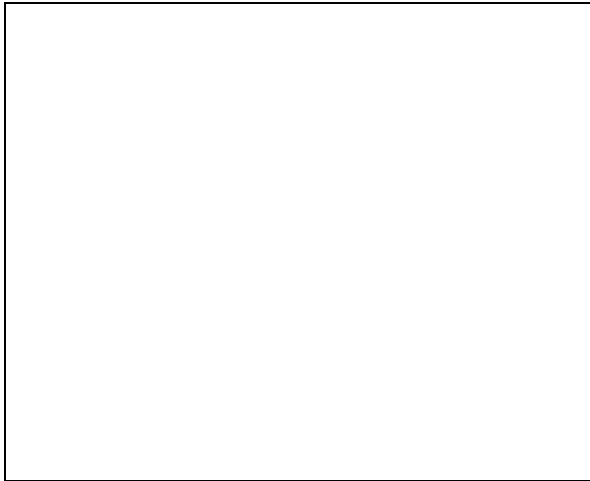


Name: _____

14) Write the exponential equation you have stored as Y3.

15) Press $\boxed{Y=}$ and move the cursor over the = in the Y2 equation. Press $\boxed{\text{ENTER}}$. Repeat this process for Y3 so that the = in the Y3 equation is the only one highlighted. Press $\boxed{\text{GRAPH}}$.

Sketch your graph in the box below.



16) Based on these graphs, what equation do you feel is the best fit for the data? Why?

17) Examine the three graphs again using $\boxed{\text{ZOOM}}\boxed{6}$ to select [ZSTANDARD]. Do you still feel the same model is the best fit? Why or why not?

Name: _____

- 18) If you were able to generate more data in your experiment, so that you had more bounces on the left side of graph. Where would the peak points lie? Would they fit the model you chose? Explain your answers.
- 19) If the CBR had gathered data for 30 seconds instead of only 15, what effect would that have on your data and your graphs. Does the model you picked fit those expectations? Explain your answers.
- 20) At what point during this activity did you become convinced that an exponential model was the best non-linear model for this data?

Name: _____

21) Explain, in plain language, what makes linear, quadratic, and cubic models inappropriate models to describe the relationship of ball height and time of bounce peak.

22) What is an error that other students might make when attempting this activity? Why might they make that error and where would the first be likely to recognize that something was wrong? How would they know?